

Logic and Justice

An Introduction to Propositional and First-Order Logic,
with an Emphasis on their Relevance to Political Reform

Isaac Wilhelm

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*When logic and proportion
have fallen sloppy dead,
and the White Knight is talking backwards
and the Red Queen's "Off with her head!"
remember what the dormouse said:
"Feed your head! Feed your head!"*

— Grace Slick

Chapter 1

Introduction

Some arguments are good and other arguments are bad. We are all familiar with bad arguments. Discussions of politics, race, gender, and economics, are littered with them. And we generally consider our own arguments to be good. Our arguments for particular views about various hot-button topics are usually successful; or so we think.

But what distinguishes good arguments from bad ones? What must an argument be like—what properties must it have—in order to be good? What, in short, makes some arguments good and other arguments bad?

These questions are not mere academic, ivory-tower speculation. They are extremely important, and their answers really matter. For in arguments, as in life, not just anything goes. Some arguments really do establish their conclusions; other arguments really do not. Some arguments, that is, are good; others, not so much. And in order to change the world for the better—to promote social justice, enact political reform, and so on—we need a theory of the distinction between good arguments and bad ones.

This book is an introduction to two such theories: propositional logic, and

first-order logic. Both propositional logic and first-order logic consist of a formal language, an account of what it takes for sentences in that language to be true, and an account of what it takes for arguments in that language to be good.¹ These resources can be used to formulate a reasonably decent theory of what makes some English arguments good and other English arguments bad. The purpose of this book is to explain exactly how.

Here is a rough outline of the theory to come. Roughly put, an argument is good just in case it can be translated into a formal argument—in either the language of propositional logic, or the language of first-order logic—which has the following property: every way of making the premises true is also a way of making the conclusion true. Good arguments, in other words, are arguments in which the truth of the premises *guarantees* the truth of the conclusion. Propositional logic provides one precise account of what it means for a conclusion to be ‘guaranteed’ by some premises. First-order logic provides another, complementary account.²

By the end of this book, you will be able to take an English argument—from a facebook post, a tweet, a video clip, or wherever—and

(i) extract a more rigorous English argument from it,

¹There are many different ways in which an argument can be good, of course. Some arguments are good in the sense that they are convincing. Other arguments are good in the sense that they establish morally good views. In this book, I focus on one particular way in which arguments can be good: they can be good in the sense of having a good formal structure, that is, a structure of the sort that propositional logic—and first-order logic—describe. These different ways of being good are not independent of one another. They are extremely intertwined: arguments which have a good formal structure, for instance, are often more convincing than arguments which do not. So to understand everything that goes into good arguments, we need to improve our understanding of all the different ways in which arguments can be good. And to do that, we need to understand the particular way of being good that will be discussed in this book.

²There are more accounts of how arguments can be good. Those accounts are based on logical systems which will not be discussed here. For presentations of some such systems, see (Priest, 2008; Sider, 2010). In this book, I focus on propositional logic and first-order logic because they are perhaps the simplest, strongest, and most influential accounts of the distinction between good arguments and bad arguments. So they are among the most important accounts to study.

- (ii) translate that more rigorous argument into either the language of propositional logic or the language of first-order logic,
- (iii) use propositional logic, or first-order logic, to determine whether that translated argument is good or bad, and then
- (iv) determine, on the basis of that, whether or not the original English argument is good or bad.

Taken together, steps (i)–(iv) represent a simple, accessible, yet rigorous theory of the distinction between good arguments and bad arguments. By learning how to follow steps (i)–(iv), you will learn how to draw that distinction.

As a result, you will be better equipped to evaluate arguments in articles, papers, books, blog posts, social media, advertisements, video clips, television, movies, and more. Steps (i)–(iv), in other words, will help you accurately assess the arguments that you routinely encounter. You will be able to tell when someone gives you an argument that fails to establish its conclusion. You will be able to tell when the assortment of claims that someone makes, passionately and perhaps charismatically, do not actually form an argument at all. And your capacity for constructive self-criticism will improve: you will be better at determining when you have, and when you lack, good arguments for your views. So in general, after learning steps (i)–(iv), you will be a more effective, clear-headed, and honest reasoner.

And that, in turn, will make you a more effective agent for social change. Of course, to achieve justice, we need more than just good reasoners: we need better politicians, more effective schools, improved health care, and many other things. But we need good reasoners too. We need people who can effectively evaluate the arguments given by politicians, lawyers, lobbyists, CEOs, billionaires, journalists, media personalities, religious leaders, and others in positions of power. And logic can help with that.

A quick note on the scope and limits of this book. Many introductory logic books cover more details than I will cover here.³ Such books are great for those who want to delve into the abstract theory of logic, perhaps because they want to become logicians, or computer scientists, or linguists, or philosophers. But such books are not so great for those who just want to use logic to improve their everyday reasoning about things that matter to them. Similarly, many critical reasoning books cover more social justice issues—and cover them more deeply—than I will here.⁴ Such books are great for those who want a big-picture, birds-eye overview of logic and reasoning. But such books are not so great for those who want to learn, in rigorous detail, the core components of logic itself.

This book helps fill the gap between these two literatures. On the one hand, it contains more detailed discussions of the relevance of formal logic to everyday life—the connection, in particular, between logic and social justice—than standard logic textbooks. On the other hand, it contains more detailed discussions of formal logic than standard critical reasoning books.

Here is a summary of the material to come. Part I covers the basic theory of natural language arguments. In particular, in Chapter 2, I explain what natural language arguments are. Then I explain—in a rough and intuitive sense—what makes them good or bad.

Part II covers the core theory of propositional logic. In Chapter 3, I introduce the formal language. I also explain how that language can be used to translate English sentences. In Chapter 4, I give a fully rigorous account of truth in propositional logic. Then I give a fully rigorous account of what

³For other introductions to propositional logic and first-order logic, as well as discussions of those logical systems' more advanced features, see (Chiswell & Hodges, 2007; Enderton, 2001; Shoenfield, 1967).

⁴For examples, see (Burgis, 2019; Cheng, 2018; Linker, 2015; Stebbing, 1939; Zornado et al., 2020).

it takes for an argument in propositional logic to be good. In Chapter 5, I use ideas from Chapter 3 and Chapter 4 to precisify the ideas from Part I. In particular, I use the account of good arguments in propositional logic to sharpen the account of what makes natural language arguments good. Finally, in Chapter 6, I explain how to use these ideas from propositional logic to evaluate the sorts of arguments that appear in social media, news articles, sound bites, radio shows, and so on.

Part III covers some shortcomings of propositional logic. In particular, in Chapter 7, I show that the theory presented in Part II is—though pretty good—not ideal. That theory classifies some perfectly good English arguments as bad. To correctly classify those arguments as good, propositional logic will not cut it. A better logical system is needed.

Part IV covers the core theory of that better system: first-order logic. In Chapter 8, I introduce the formal language. I also explain how that language can be used to translate English sentences. In Chapter 9, I give an account of truth in first-order logic. Then I give an account of what it takes for an argument in first-order logic to be good. Unfortunately, in practice, this account of good arguments is hard to use. So in Chapter 10, I present another account of what it takes for an argument in first-order logic to be good. This other account (i) agrees with the original account, in that they both say that exactly the same arguments are good, but (ii) is much easier to use. In Chapter 11, I use the ideas from Chapter 8, Chapter 9, and Chapter 10, to precisify the ideas from Part I. In particular, I use the account of good arguments in first-order logic to sharpen the account of what makes natural language arguments good. And I explain how these ideas from first-order logic can be used to evaluate the sorts of arguments that you routinely encounter.

Part V contains a review of the entire book. Appendix A summarizes the

content of each chapter. Appendix B contains a list of all the key definitions.

Part I

Natural Language Arguments

Chapter 2

Arguments in English

In this chapter, I introduce the basic idea of an argument in English. To start, I briefly explain—in untechnical terms—what arguments are. Then I define some important notions related to arguments. After that, I discuss two ways in which arguments can be good: they can be valid, and they can be sound. Finally, I explain why it is worth thinking about arguments at all.

2.1 Arguments, Reasons, and Views

Very roughly put, an argument consists of (i) a view, and (ii) some reasons for that view. In a good argument, the view is supported by the reasons. In a bad argument, the reasons do not support the view. But in every argument, good or bad, there is a view and there are some reasons for it. Arguments, in short, are reasons for views.

For example, consider the passage below.

The Civil Rights Act helps people. For after all, if the Civil Rights Act outlaws racial discrimination, then it is extremely helpful to our society as a whole. And the Civil Rights Act does just that: it outlaws discrimination on the basis of many things, including race. So it is extremely helpful.

This passage contains an argument. The view is that the Civil Rights Act helps people. The reasons are (i) the Civil Rights Act outlaws racial discrimination, and (ii) if the Civil Rights Act outlaws racial discrimination, then the Civil Rights Act helps people. Those reasons, together with that view, comprise the argument that the above passage contains.

When an argument is expressed in the form of a written paragraph—as in the passage above—it can be difficult to determine the view and the reasons. There are dozens of little tricks which one might use, to determine all that. For instance, words like ‘so’ and ‘therefore’ typically signal that whatever follows is a view, while words like ‘for’ and ‘because’ typically signal that whatever follows is a reason. But there are no hard-and-fast rules for determining, for any given passage, which parts of that passage express views, which parts of that passage express reasons, and which parts of that passage express neither. Mostly, it just takes practice.¹

Once an argument has been extracted from a passage, it often helps to rewrite that argument in the format of a list. For example, here is an especially clear way to express the argument in the passage above.

¹It also helps, a great deal, to know propositional logic and first-order logic. Both logical systems can be used to re-express natural language arguments—like the argument about the Civil Rights Act—in more formal, structured, logically rigorous ways. And that, in turn, is helpful for figuring out, for any given passage, which parts of that passage express views and which parts of that passage express reasons.

The Civil Argument

1. The Civil Rights Act outlaws racial discrimination.
2. If the Civil Rights Act outlaws racial discrimination, then the Civil Rights Act helps people.
3. The Civil Rights Act helps people.

I call this the ‘Civil’ argument. The first two lines, in the Civil argument, are the reasons. The last line, the Civil argument, is the view.

For the most part, in this book, I focus on arguments written in the form of the Civil argument above. That is, I focus on arguments expressed as a list of reasons followed by the view. Call these ‘numbered’ arguments, since they are expressed by numbered lists of reasons and views.

Numbered arguments probably seem pretty artificial. Usually, when we give an argument in everyday life, we do not put the reasons and the view into a numbered list. We sort of ‘smoosh’ it all together, into a single paragraph, as in the passage about the Civil Rights Act from before. Call these ‘colloquial’ arguments, since they are expressed colloquially, in a kind of uncomplicated, familiar, everyday sort of way.

In Chapter 6, I say a bit more about how to extract numbered arguments from colloquial arguments. That process, of argument extraction, makes crucial use of propositional logic. Basically, propositional logic—and first-order logic too—can help inform your guesses as to which parts of passages are views and which parts of passages are reasons. So before seeing more examples of how argument extraction works, it is worth learning the basics of propositional logic. And to do that, it is easiest to focus on numbered arguments, at least for a while.

2.2 The Basics of Natural Language Arguments

In this section, I define three important notions. The first is that of an English-language argument.² The second is that of an argument’s conclusion. The third is that of a premise in an argument.

To start, here is a preliminary definition. An ‘English proposition’ is, roughly, an English sentence which is either true or false. Such sentences are often called ‘truth-evaluable’: their truth values, that is, can be evaluated. For example, the sentence “The Civil Rights Act outlaws racial discrimination” is true. So that sentence is truth-evaluable; that sentence, in other words, has a truth value. And so that sentence expresses a proposition.

Now for the definition of an argument. As you will see, it is not very complicated.

Definition 2.1: Argument (in English)

An ‘argument’ in English is a sequence of two or more English propositions.

We have already seen an example of an argument in English: the Civil argument, reproduced below.

1. The Civil Rights Act outlaws racial discrimination.

²I focus on English arguments for two reasons. First, I am pretty good at speaking English. I am not very good, however, at speaking any other natural languages. Second, this book is designed to help as many people as possible in the contemporary U.S. Most of those people are English-speakers; this book was written so as to be accessible to them. But to be clear: there is nothing particularly special about English. For the purposes of writing a book on the relationship between natural language and logic, most any other natural language—Navajo, Spanish, Mandarin, Hindi, Arabic, Swahili, and so on—could be used instead.

2. If the Civil Rights Act outlaws racial discrimination, then the Civil Rights Act helps people.
3. The Civil Rights Act helps people.

Each of these three lines is a proposition, since each line is either true or false. So these three lines constitute a sequence of propositions. In addition, this sequence is more than two lines long: it consists of three lines. So it is an argument (in English).³

As another example, consider the argument below.

The Company Argument

1. All companies should permit the formation of unions.
2. Amazon is a company.
3. Amazon should permit the formation of unions.

This is an argument too: each line is either true or false, and there are at least two lines in the above sequence. I will call it the ‘Company’ argument.

Here is an example of a non-argument.

Non-Argument

³Some sequences of English propositions are not presented as arguments. For instance, suppose we number the first three lines of “A Christmas Carol,” as follows.

1. Marley was dead, to begin with.
2. There is no doubt whatever about that.
3. The register of his burial was signed by the clergyman, the clerk, the undertaker, and the chief mourner.

These three sentences are propositions, so by Definition 2.1 above, they form an argument. But in the context of “A Christmas Carol,” they are not presented as such. They are presented, instead, as part of a story. So whether a given sequence of propositions is an argument depends, at least to some extent, on the context in which it is presented. For the purposes of this book, however, I will set this issue—concerning the contextual sensitivity of arguments—aside. To keep things simple, in what follows, I take any sequence of two or more propositions to be an argument.

1. Give to charity.
2. If you give to charity, then you will help people.
3. You will help people.

This is not an argument because the first line is not a proposition. To see why, just note that the first line is neither true nor false. “Give to charity” is not truth-evaluable: it is a command. So it cannot be false and it cannot be true.

Now for the definitions of conclusions and premises. Roughly put, the conclusion of an argument is that argument’s view. And roughly put, the premises of an argument are that argument’s reasons. The terms ‘conclusion’ and ‘premise’ are, in other words, the technical counterparts of the terms ‘view’ and ‘reason’ from Section 2.1.

In numbered arguments, conclusions and premises may be defined quite precisely. For recall that the last line, of a numbered argument, is that argument’s view. And recall that the other lines, of a numbered argument, are that argument’s reasons. So conclusions and premises, for our purposes here, may be defined as follows.

Definition 2.2: Conclusion (English argument)

The ‘conclusion’ of an argument in English is the last proposition in that argument.

Definition 2.3: Premise (English argument)

A ‘premise’ of an argument in English is any proposition in that argument which is not the conclusion.

For example, in the Civil argument, line 3 is the conclusion. Line 1 and line 2

are premises.

Here are three more examples of arguments.

Argument

1. Either the Civil Rights Act passed in 1931, or the Civil Rights Act passed in 1964.
2. The Civil Rights Act did not pass in 1931.
3. The Civil Rights Act passed in 1964.

Argument

1. Jan Morris wrote *Conundrum*.
2. Virginia Woolf wrote *To the Lighthouse*.
3. Sui Hui wrote *Star Gauge*.
4. Jan Morris wrote *Conundrum*, Virginia Woolf wrote *To the Lighthouse*, and Sui Hui wrote *Star Gauge*.

Argument

1. Some fetuses are people.
2. All people have a right to life.
3. Some fetuses have a right to life.

An argument can have any finite number of lines. My examples generally feature just three lines, because three-line arguments are particularly simple and easy to discuss. But arguments can have four lines—like the second argument above—or five lines, or more.

Here is another example of a non-argument.

Non-Argument

1. If we should reform the justice system, then I will support reformation.
2. Should we reform the justice system?
3. I will support reformation.

This is not an argument because line 2 is not a proposition. Line 2 is a question, and questions are not the sorts of things that are true or false.

Here is another example.

Non-Argument

1. Ouch!
2. Either getting punched hurts, or getting punched does not hurt.
3. Getting punched hurts.

This is not an argument, since line 1 is not a proposition.

Here are two more examples of arguments. They feel like non-arguments. But they are arguments nonetheless.

Argument

1. Muhammad was Muslim.
2. Moses was Jewish.
3. Paul was Christian.

Argument

1. If Ed Roberts studied at Berkeley, then Ed Roberts studied at a famous college.
2. Patrisha Wright lobbied for the ADA.
3. More buildings should have ramps and elevators, in addition to stairs.

Despite appearances, these are indeed arguments. For they are sequences of two or more propositions. They are just *bad* arguments; that is why they sound bizarre. So they do count as arguments; they are merely terrible ones. In the next section, I discuss what makes some arguments good and other arguments bad.

2.3 Validity and Soundness

There are many different ways in which an argument can be good. In this book, I will focus on two: arguments can be good by being valid, and arguments can be good by being sound.

Briefly put, valid arguments are good because they have a very nice form. The premises and the conclusion all ‘fit’ together. Below, I define this special kind of ‘fit’ a bit more precisely.⁴ Sound arguments are good because (i) they are valid, and (ii) they also have true premises. So these arguments have a nice form, but they have an additional virtue: their premises are all true.

Now for first-pass definitions of valid arguments, invalid arguments, sound arguments, and unsound arguments. To be clear: these will not be our final definitions of those notions. Final definitions will not be presented until we

⁴One fully precise definition will be provided in Chapter 5. Another fully precise definition will be provided in Chapter 10.

have learned a lot more logic. Rather, these definitions are intended to capture the intuitive, squishy, but still extremely important notions of validity, invalidity, soundness, and unsoundness for natural language arguments. So I call these ‘preliminary’ definitions.

To start, here is the preliminary definition of a valid argument in English.

Definition 2.4: Valid Argument (in English, preliminary)

A ‘valid argument’ in English is an argument in English which has the following property: if the argument’s premises are all true, then the argument’s conclusion must also be true.

To put it another way, in a valid argument, the truth of the premises *guarantees* the truth of the conclusion. This is what I meant earlier, when I wrote that in a valid argument, the premises and the conclusion all ‘fit’ together. They ‘fit’ together in the sense that the premises’ truth guarantees the conclusion’s truth.

The Civil argument—reproduced below—is valid. To see why, let us look at it more closely.

1. The Civil Rights Act outlaws racial discrimination.
2. If the Civil Rights Act outlaws racial discrimination, then the Civil Rights Act helps people.
3. The Civil Rights Act helps people.

To check that this satisfies Definition 2.4, suppose that lines 1 and 2—the premises—are true. Suppose, in other words, that (1) the Civil Rights Act does indeed outlaw racial discrimination, and (2) *if* the Civil Rights Act does

that, *then* the Civil Rights Act helps people. From those two assumptions, it follows that the Civil rights act helps people. That is, line 3—the conclusion—holds. So if the premises are true, then the conclusion must be true. And so the Civil argument is valid.

The Company argument is valid as well. To see why, consider it once more.

1. All companies should permit the formation of unions.
2. Amazon is a company.
3. Amazon should permit the formation of unions.

To check that this satisfies Definition 2.4, suppose that lines 1 and 2—the premises—are true. Suppose, in other words, that (1) companies really should allow unions to be formed, and (2) Amazon is a company. From those two assumptions, it follows that Amazon should permit the formation of unions. That is, line 3—the conclusion—holds. So if the premises are true, then the conclusion must be true. And so the Company argument is valid.

Before continuing to explore examples, it is worth introducing a few more definitions. Here is the preliminary definition of an invalid argument in English.

Definition 2.5: Invalid Argument (in English, preliminary)

An ‘invalid argument’ in English is an argument in English which is not valid.

So an argument is invalid just in case that argument’s premises could all be true, and yet the conclusion could nonetheless, still, be false.

Here is the preliminary definition of a sound argument in English.

Definition 2.6: Sound Argument (in English, preliminary)

A ‘sound argument’ in English is a valid argument in English whose premises are all true.

In other words, in order for an argument to be sound, it must be (i) valid, and (ii) have true premises.

Finally, here is the preliminary definition of an unsound argument in English.

Definition 2.7: Unsound Argument (in English, preliminary)

An ‘unsound argument’ in English is an argument in English which is not sound.

So an argument is unsound just in case either (i) it is invalid, or (ii) at least one of its premises is false.

Earlier, I claimed that validity and soundness represent two important ways for an argument to be good. So if you want your arguments to be good ones—and of course, you should want that—then your arguments should be valid and sound. Unsound, invalid arguments are bad.

One might wonder: why? What is it about valid arguments that makes them so good? What is it about sound arguments that makes them so good too? Why are invalid arguments bad? And why are unsound arguments bad?

Here is one of the main reasons why valid arguments are valuable: if you have a valid argument for a view, then in order to establish that view, you need only establish that argument’s premises. For if your argument is valid, then the truth of the premises *guarantees* the truth of the conclusion. So if

your premises hold, then your view must hold too. That is simply what it is, for an argument to be valid.

There is another, related reason why valid arguments are good. Suppose you have a valid argument for a view. And suppose that someone else disagrees with the view in question: they think, for whatever reason, that the view in question is false. Then this other person is committed to thinking that at least one of your premises is false as well. For again, in a valid argument, if the premises are all true then the conclusion must be true. So this other person cannot accept all of the premises, if they want to reject the conclusion. And so if they want to disagree with you, they must do more than say that your conclusion is false. They must say that one of your premises is false as well.

An example will help explain why this is so helpful. Suppose that Skyler and Britney are debating the Civil Rights Act. Skyler thinks that the Civil Rights Act helps people. Brittany, however, does not. So Skler presents Britney with an argument: the Civil argument from earlier. The Civil argument is valid, so if its premises are all true, then its conclusion must be true. Therefore, in order to reject the conclusion, Britney must also reject one of the premises. For if Britney is right that the conclusion is false, then because the argument is valid—because, in other words, the premises’ truth guarantees the conclusion’s truth—one of the premises must be false as well. In other words, Britney cannot simply keep insisting on the falsity of Skyler’s conclusion. That is no longer sufficient, to reject what Skyler is saying. For if Skyler’s premises hold, then Skyler’s conclusion does too. So Britney must reject one of the premises, if she wants to continue claiming that the Civil Rights Act does not help people.

In this way, valid arguments help us get beyond merely insisting that this view is true and that view is false. Valid arguments help us see what we must accept, if we accept certain premises. They help us see what we must

reject, if we reject certain conclusions. It would be unproductive for Sklyer to repeatedly assert, and for Britney to repeatedly deny, that the Civil Rights Act helps people. It is much more productive for one of these people—Sklyer, say—to formulate a valid argument for their view. For once they do, then the other person must do more than disagree with the view in question. The other person must say which premise, in the valid argument, is false. And so the debate can shift to something more productive than just disagreeing over a view. It can shift to an examination of the reasons for that view, to see whether or not some of those reasons are false.

Sound arguments are valuable in a similar way. If there is a sound argument for a view, then that view *must* be true. For if an argument is sound, then the premises are true and the argument is valid. So by the definition of validity, the conclusion is guaranteed. And so if you have a sound argument for some claim, then the claim holds. It is worth emphasizing this: the claim *absolutely must* hold, if the argument is sound. No one can truthfully deny it. For again, if the argument is sound, then (i) the premises are true, and (ii) the argument is valid; and so by the definition of validity, (iii) the conclusion must be true as well. Therefore, if you have a sound argument for some claim, then the truth of the claim is guaranteed. It does not matter what anyone says, or thinks, or wants. The claim is simply true.

This is why logic is one of the most important subjects to study. Propositional logic, and first-order logic, are theories of validity and soundness: they say, in more detail than I did above, exactly what validity and soundness are. And as was just explained, when reasoning, we should use valid and sound arguments. So the world would be a much better place, if people reasoned in the ways that propositional logic and first-order logic suggest. Not because people would be robots; obviously, they would not be. The world would be

a better place because people would be clearer thinkers, and people would be harder to mislead.

It is extremely, extremely important to be a good reasoner. If someone is bad at reasoning, then it is very easy to lead them astray. Politicians take advantage of this all the time, by lying, bullshitting, and so on. In fact, what many politicians do is

- (i) say things that seem like arguments, but are actually non-arguments,
- (ii) give arguments which seem good, but are actually invalid,
- (iii) give arguments which—though valid—have hidden false premises, and so are unsound.

By studying logic, you will become better at detecting when politicians try to take advantage of you in these ways. Of course, there are many other ways in which politicians take advantage of us. But this is an extremely important one. And it is one that you can, pretty easily, do something about: just learn some logic.

It is worth seeing more examples of valid, invalid, sound, and unsound arguments. To start, here is an example of an argument—mentioned before—which is both valid and sound.

Argument (valid, sound)

1. Either the Civil Rights Act passed in 1931, or the Civil Rights Act passed in 1964.
2. The Civil Rights Act did not pass in 1931.
3. The Civil Rights Act passed in 1964.

This argument should ‘feel good’ to you. For intuitively, it does seem to be

the case that if both premises are true, then the conclusion must be true. So this argument is, intuitively, valid. In addition, both premises are in fact true. So this argument is sound.

Here is another example.

Argument (valid, sound)

1. Jan Morris wrote *Conundrum*.
2. Virginia Woolf wrote *To the Lighthouse*.
3. Sui Hui wrote *Star Gauge*.
4. Jan Morris wrote *Conundrum*, Virginia Woolf wrote *To the Lighthouse*, and Sui Hui wrote *Star Gauge*.

This argument is valid because if the premises are all true, then the conclusion must be true. In addition, this argument is sound because (i) it is valid, and (ii) the premises all hold.

The argument below is valid, and may or may not be sound.

Argument (valid, maybe sound)

1. Some fetuses are people.
2. All people have a right to life.
3. Some fetuses have a right to life.

This argument is valid: if some fetuses really are people, and if every person really has a right to life, then it follows automatically that some fetuses have a right to life too. Whether or not this argument is sound, however, remains extremely controversial in the contemporary U.S. For while the truth of line 2

is more-or-less generally accepted, line 1 is the subject of much debate. Some claim that all fetuses are people, some claim that only fetuses of a certain age are people, and some claim that no fetuses are people at all. The soundness of this argument depends, ultimately, on whether line 1 is true; and at the moment, there is much disagreement about that.

Here is an example of an invalid, and so unsound, argument.

Argument (invalid, so unsound)

1. Either the Civil Rights Act passed in 1964, or the Civil Rights Act passed in 1970.
2. The Civil Rights Act passed in 1964.
3. The Civil Rights Act passed in 1970.

This argument is not valid, for the following reason: there is a way for the premises to be true without the conclusion also being true. To see how, suppose that the Civil Rights Act passed in 1964 and not in 1970. That certainly seems possible; in fact, it is actually true. And if that is the case, then line 2 is true. Line 1 is true as well: if the Civil Rights Act did indeed pass in 1964, then it is true that the Civil Rights Act passed either in 1964 or in 1970. But line 3 is false: the Civil Rights Act did not pass in 1970. So both of the premises can be true without the conclusion also being true. In other words, if the premises are true, it is *not* the case that the conclusion *must* be true. Therefore, this argument is invalid. And because of that, this argument is unsound.

Here is another example of an invalid, and so unsound, argument.

Age Argument (invalid, so unsound)

1. Some twenty-year-olds are Republicans.

2. Some Republicans are sixty years old.
3. Some twenty-year-olds are sixty years old.

The Age argument is invalid. To see why, note that the premises are true. Some twenty year old individuals are indeed Republicans; so line 1 holds. And some Republicans are indeed sixty years old; so line 2 holds. But obviously, line 3 is false: no twenty-year-olds are sixty. Therefore, this argument is invalid. And so this argument is unsound too.

The following argument illustrates an important feature of invalidity: some invalid arguments have true premises and also a true conclusion. For example, consider the argument below.

The Non-Overlap Argument (invalid, so unsound)

1. Some feminists are conservatives.
2. Some conservatives are pro-life.
3. Some feminists are pro-life.

The Non-Overlap argument might seem valid, because all three lines are actually true. Some feminists are indeed conservatives, some conservatives are indeed pro-life, and some feminists are indeed pro-life as well. Nevertheless, the Non-Overlap argument is invalid, because the truth of the premises does not *guarantee* the truth of the conclusion. In other words, even if the premises are all true, it does not *automatically follow* that the conclusion *must* be true. To see why, suppose that some feminists are conservatives, and suppose that some conservatives are pro-life. So lines 1 and 2 are true. Even so, it is *possible*—not actual, but at least possible—that line 3 is false: it is possible that no feminists are pro-life.

To see what is going on here, consider the following picture.

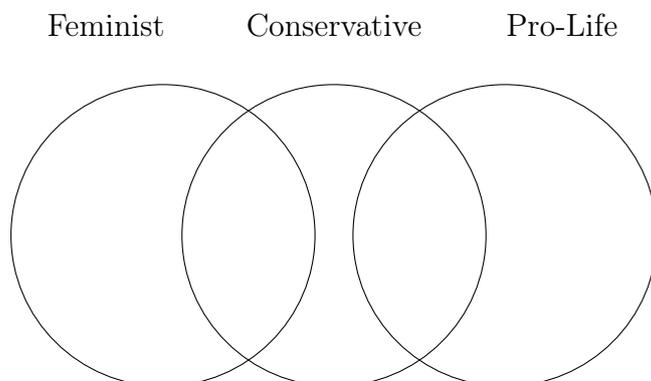


Figure 2.1: this picture explains the invalidity of the Non-Overlap argument.

The circles represent properties: the circle below the word ‘Feminist’ represents the property of being a feminist, the circle below the word ‘Conservative’ represents the property of being a conservative, and the circle below the word ‘Pro-Life’ represents the property of being pro-life. The points in any given circle represent the people with that circle’s property. For instance, the points in the circle below ‘Feminist’ represent people who are feminists.

Figure 2.1 represents a *possible*—not actual, but possible—state of affairs.⁵ In this state of affairs, the ‘Feminist’ circle overlaps the ‘Conservative’ circle. So some feminists are conservative. And in this state of affairs, the ‘Conservative’ circle overlaps the ‘Pro-Life’ circle. So some conservatives are pro-life. But in this state of affairs, the ‘Feminist’ circle does not overlap the ‘Pro-Life’ circle. So no feminists are pro-life.

Since Figure 2.1 represents a possible state of affairs, the Non-Overlap argument is invalid. For figure 2.1 demonstrates a way in which the premises of that argument could all be true while the conclusion is false. So the truth of

⁵The actual state of affairs, regarding feminists and conservatives and being pro-life, would look like a Venn diagram with three circles. That is, a picture which represented how things actually are would not look like 2.1: it would allow the Feminist circle and the Pro-Life circle to overlap.

the premises, in the Non-Overlap argument, does not guarantee the truth of the conclusion. It does not matter that as a matter of actual fact, all three lines of the Non-Overlap argument are true. Since it is possible for the premises to be true while the conclusion is false, the Non-Overlap argument is invalid. And because of that, the argument is unsound as well.

Because of all this, the Non-Overlap argument is invalid for exactly the same reason as the Age argument. The reason, in both cases, is this: the truth of the premises, of the argument in question, fails to guarantee the truth of that argument's conclusion. In the case of the Age argument, this is quite easy to see. The premises of the Age argument are actually true, and the conclusion of the Age argument is actually false. So obviously, the premises' truth does not guarantee the conclusion's truth. In the case of the Non-Overlap argument, however, things are a bit more confusing. For in the Non-Overlap argument, the premises and the conclusion are all true. The argument is still invalid, however, because it is *possible* for the premises to be true while the conclusion is false; that is what Figure 2.1 shows. So in the Non-Overlap argument, as in the Age argument, the premises' truth does not guarantee the conclusion's truth; and so the Non-Overlap argument is invalid.

Here is an example of something which is neither valid, nor invalid, nor sound, nor unsound.

1. Give to charity.
2. If you give to charity, then you will help people.
3. You will help people.

Recall that, as discussed before, this is not an argument at all. So this sequence of sentences does not satisfy any of the definitions—of validity, inva-

lidity, soundness, or unsoundness—given above.

Many arguments are valid but not sound. Here is an example.

Argument (valid, unsound)

1. If trickle-down economic theory is true, then we should lower taxes for the rich.
2. Trickle-down economic theory is true.
3. We should lower taxes for the rich.

This argument is valid. If both premises are true, then clearly, the conclusion must follow. But it is not the case that both premises are true. Line 2 is false:⁶ as numerous economic models have shown, trickle-down economic theory—the version, at least, discussed most by politicians and the media—does not hold.⁷ So this argument is unsound.

The arguments above are, for the most part, pretty simple. Because of that, they are not the sorts of arguments which tend to arise in everyday life. So in closing, here are two examples of more complicated arguments. These are similar to arguments which you might have encountered in social media, magazines, conversations with others, and so on. The first argument is about defunding the police.

The Defunding Argument

⁶Arguably, line 1 is false as well. Even if trickle-down economic theory were true, it could still be false that the rich ought to pay less in taxes. There may well be other reasons for the rich to pay as much as—or more than—they currently do.

⁷Trickle-down economics, as understood in popular culture, is not really much of an economic theory at all. Supply-side economics, to which trickle-down economics is closely related, represents something a bit more akin to a general economic theory. See (Wanniski, 1998) for a general account of that theory; see (Quiggins, 2010) for accessible criticisms of it. For an example of the sorts of rigorous economic studies which undermine the theory of supply-side economics, see (Goolsbee, 1999).

1. The police are not trained for many tasks that they often end up doing.
2. Anti-bias training for police does not work.
3. When the police screw up, they are often not held accountable.
4. If police are not trained for many tasks that they often end up doing, and anti-bias training for police does not work, and it is the case that if the police screw up then it is hard to hold them accountable, then we should defund the police.
5. We should defund the police.

The second argument is about gun ownership.

The Firearms Argument

1. Firearms can save lives.
2. Some firearms are not used in mass shootings.
3. Anything can save lives, yet is not used in mass shootings, should be legal to own.
4. It should be legal to own some firearms.

Since these arguments are somewhat complex, it may be hard to determine—merely by looking at them—whether or not they are valid. Even after rereading these arguments a few times, you may still be unsure about their quality.

That is another reason why logic is worth studying. For as will become clear, propositional logic and first-order logic can help you evaluate arguments like these. As I mentioned above, definitions 2.4–2.7 are merely rough, first-pass, intuitive definitions of validity, invalidity, soundness, and unsoundness. Propositional logic, it turns out, has the resources to formulate an extremely

precise definition of validity; and so by extension, of invalidity, soundness, and unsoundness as well. Similarly, first-order logic has the resources to formulate extremely precise definitions of validity, invalidity, soundness, and unsoundness. In fact, propositional logic and first-order logic were designed, in large part, for that very purpose. So that is another reason why propositional logic and first-order logic are worth learning: they can help you evaluate arguments that are too complicated to be evaluated informally.

For example, propositional logic and first-order logic can be used to evaluate the two complicated arguments given above. In particular, propositional logic can be used to determine whether or not the Defunding argument is valid; for an explanation of exactly how, see Chapter 6. And first-order logic can be used to determine whether or not the Firearms argument is valid; for an explanation of exactly how, see Chapter 11.

2.4 The Value of Arguments

One might wonder: why care about arguments at all? Why do arguments matter? What is the point of offering arguments for your views? What is the point of listening to arguments for views with which you disagree? Why, in short, are arguments valuable?

Arguments are valuable in many, many different ways; here, I list four. First, arguments against your views can help you correct any bad views which you might hold. Everyone, of course, has false beliefs: I do, you do, and so does everyone that you know. Arguments can help us figure out what those false beliefs are, and what corrections need to be made. So when someone offers you a considerate, thoughtful argument, they are probably doing you

a favor. They are giving you an opportunity to see if your views are worth holding. And it is often worth taking advantage of that opportunity.

Second, by offering good arguments in favor of your views, you can help improve other peoples' beliefs. Again, everyone has beliefs which are false. By offering arguments for your views, you can help others figure out what those false beliefs are. So when you give someone a considerate, thoughtful argument, you are doing them a favor; just as they are doing you a favor, when they offer you an argument like that. You are giving them an opportunity to reevaluate their views, and see if those views are worth endorsing.

Third, some arguments—when made with compassion, kindness, and care—can help us improve our relationships. When you argue thoughtfully with a friend, an acquaintance, a partner, or a stranger, you build your relationship with that person. You learn about the other person, about yourself, and about the connection which the two of you share. Of course, sometimes, even compassionate, kind, and thoughtful arguments can cause trouble for a relationship. But very often, when offered in a spirit of thoughtfulness and open-mindedness, arguing can make a relationship stronger. It can deepen trust and build intimacy. It can also make you a wiser person. For it can help you become a better communicator, listener, speaker, and thinker.

Fourth, arguments help us better understand the world. After hearing a successful argument for a view, we can get a better sense for why that view might hold; or at least, for why we should consider adopting it. After hearing an unsuccessful argument for a view, we can get a better sense for why that view might not hold; or at least, for why we should consider giving it up. Arguments, in short, can teach us how various facts, assumptions, posits, and so on, fit together. And because of that, arguments can teach us about the nature of reality itself.

Some people claim that, when it comes to certain views at least, they should not have to argue for those views all. For instance, consider the view that all trans women are women. One might claim that there is something offensive, deeply problematic, and outright dangerous in taking this view to require argument. Trans women simply are women. To debate this is to engage in dehumanization; arguing over this is simply not permissible.

As another example, consider the view that all fetuses are children. One might claim that there is something offensive, deeply problematic, and outright dangerous in taking this view to require argument. Fetuses are simply children. To debate this is to engage in dehumanization; arguing over this is simply not permissible.

Whether it is permissible to argue over views like these depends, ultimately, on many different factors. It depends on the conversational context. It depends on the specific people who might engage in that argument. It depends on the specific societies to which those people belong. It depends on the harm that such arguing could incur. It depends on the people who could be affected. And it depends on many, many, many other factors too.

So there is no simple, strict, exceptionless rule for determining when one should, and should not, argue for a view. Arguments offer many benefits. But they can be harmful too. Overall, it can be quite difficult to determine when to argue.

My advice is: practice formulating arguments, but be kind and keep an open mind about it. Come up with arguments for your views, and come up with arguments against other views – and occasionally, when it seems appropriate, share those arguments with others. But when doing so, think about the people that your argument might affect. Think about the argument’s potential harms and potential benefits. Reflect on your motives: why are you offering this

particular argument for this particular view? Reflect on your audience: where are they coming from, and how will they hear an argument like this? Trust your instincts, but also be sure to correct your mistakes. And keep your head firmly connected to your heart.

2.5 Problems

Here are some problems based on the ideas presented in this chapter.

Problem 2.1. *Is the following an argument?*

1. *The Civil Rights Act outlaws racial discrimination.*
2. *If the Civil Rights Act outlaws racial discrimination, then the Civil Rights Act helps people.*
3. *Was Muhammad a Muslim?*

Problem 2.2. *Is the following an argument?*

1. *The Civil Rights Act outlaws racial discrimination.*
2. *If the Civil Rights Act outlaws racial discrimination, then getting punched hurts.*
3. *Sui Hui wrote Star Gauge.*

Problem 2.3. *Is the following argument valid?*

1. *The Civil Rights Act outlaws racial discrimination.*
2. *The Civil Rights Act helps people.*

3. *The Civil Rights Act outlaws racial discrimination and helps people.*

Problem 2.4. *Is the following argument valid?*

1. *The Civil Rights Act does not outlaw racial discrimination.*
2. *If the Civil Rights Act outlaws racial discrimination, then the Civil Rights Act helps people.*
3. *The Civil Rights Act does not help people.*

Problem 2.5. *Is the following argument valid?*

1. *Patrischa Wright did not attend the convention.*
2. *Either Patrischa Wright attended the convention, or George H. W. Bush did.*
3. *George H. W. Bush attended the convention.*

Problem 2.6. *Is the following argument valid?*

1. *If automatic rifles are legal to own, then you should go to the protest.*
2. *You should not go to the protest.*
3. *Automatic rifles are legal to own.*

Problem 2.7. *Give an example of a valid argument. Your example must be different from the valid arguments mentioned in this book.*

Problem 2.8. *Is the following argument sound? Why or why not?*

1. *Voguing did not originate in Brooklyn.*

2. *If voguing originated in New York, then voguing did not originate in Massachusetts.*
3. *Voguing did not originate in Massachusetts.*

Problem 2.9. *Is the following argument sound? Why or why not?*

1. *Jan Morris was not born in the U.S.*
2. *If Jan Morris was not born in the U.S., then Jan Morris was not born in New York.*
3. *Jan Morris was not born in New York.*

Problem 2.10. *Is the following argument sound? Why or why not?*

1. *If Zora Neale Hurston was born in England, then she had English citizenship.*
2. *Zora Neale Hurston did not have English citizenship.*
3. *Zora Neale Hurston was not born in England.*